Lecture 16: Self-Concordant Barriers and Primal Interior-Point Methods

TTIC 31070 / CMSC 35470 / BUSF 36903 / STAT 31015

Convex Optimization

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From Self-Concordance to Interior-Point Methods

Lecture 15 recap. Self-concordant Newton calculus on an open domain U: damped step, full-step quadratic phase, decrement certificate.

Lecture 16. Apply this to *constrained* optimization. Keep all iterates inside the feasible region; enforce the boundary through a *barrier* that blows up at ∂K.

Setup. E a finite-dim real vector space, $K \subseteq E$ closed convex with $\mathrm{int}(K) \neq \emptyset$, $c \in E^*$:

$$p^\star := \inf\{\langle c, x\rangle : x \in K\}.$$

A strictly feasible point $x_0 \in \mathrm{int}(K)$ is assumed available.

Scope. General K, general self-concordant barrier on $\mathrm{int}(K)$. **Cones** and **primal-dual** implementations $\to$ Lecture 17.

ν-Self-Concordant Barrier

Definition 16.1. $\Phi : \mathrm{int}(K) \to \mathbb{R}$ is a *ν-self-concordant barrier* for K if:

1. Φ is self-concordant on $\mathrm{int}(K)$ (Definition 15.1);
2. $\nabla^2\Phi(x) \succ 0$ for every $x \in \mathrm{int}(K)$;
3. **Boundary blow-up:** $x_j \in \mathrm{int}(K)$, $x_j \to \bar{x} \in \partial K \;\Rightarrow\; \Phi(x_j) \to +\infty$;
4. **Local gradient bound:** for every $x \in \mathrm{int}(K)$,

$$\boxed{\|\nabla\Phi(x)\|_{x,*} \le \sqrt{\nu}.}$$

Standard Barriers

Example 16.1. Two prototype barriers behind LP and SDP IPMs:

- **Nonnegative orthant** $\mathbb{R}^m_+$: $\Phi(x) = -\sum_{i=1}^m \log x_i$ on $\mathbb{R}^m_{++}$, $\nu = m$.
- **Positive-semidefinite cone** $\mathbb{S}^n_+$: $\Phi(X) = -\log\det X$ on $\{X \succ 0\}$, $\nu = n$.

Polyhedron $K = \{x : Ax \leq b\}$**.** $\Phi(x) = -\sum_{i=1}^m \log(b_i - \langle a_i, x\rangle)$. Self-concordance and $\nu = m$ follow from the orthant case + affine invariance (Prop. 15.3).

Verification of $\nu = m$ **for the orthant.** $\nabla\Phi(x) = -(1/x_1, \ldots, 1/x_m)^\top$, $\nabla^2\Phi(x) = \mathrm{diag}(1/x_i^2)$:

$$\|\nabla\Phi(x)\|_{x,*}^2 = \sum_i (1/x_i)^2 \cdot x_i^2 = m. \quad \text{Tight: } \nu = m.$$

Dikin Ellipsoid Inclusion: Local Unit Ball Lives Inside K

Lemma 16.1 (Dikin ellipsoid inclusion). Let Φ be a self-concordant barrier for K. For every $x \in \mathrm{int}(K)$,

$$\boxed{\{x + u : \|u\|_x < 1\} \;\subseteq\; \mathrm{int}(K).}$$

Why this matters. Lecture 15's *domain-safety* assumption (DS) becomes automatic. Every full Newton step with $\lambda < 1$ stays inside; every damped step with $\eta\lambda < 1$ stays inside.

Proof. Fix $x \in \mathrm{int}(K)$, u with $r := \|u\|_x < 1$. Suppose $x + u \notin \mathrm{int}(K)$. The segment $x + su$ first hits ∂K at some $\tau \in (0, 1]$.

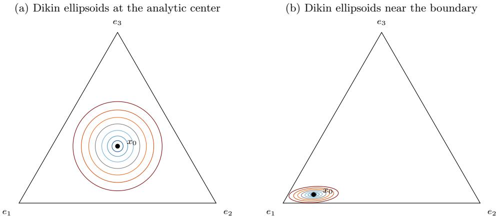
For $s < \tau$, Theorem 15.6 (self-concordant upper):

$$\Phi(x + su) \;\leq\; \Phi(x) + s\langle \nabla\Phi(x), u\rangle + \omega^*(sr).$$

Since $\tau r < 1$, RHS stays bounded as $s \uparrow \tau$. **Contradicts the boundary blow-up of Φ at $x + \tau u$.** $\square$

Two Local Geometries on the Simplex

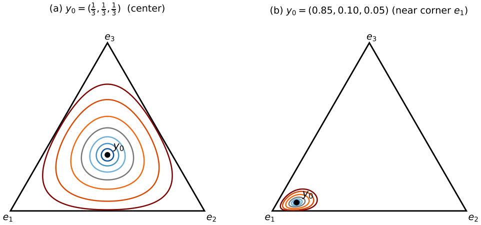
Log barrier: Dikin ellipsoids



$\Phi(x) = -\sum_i \log x_i$, Hessian metric $\|\cdot\|_x$.

Unit balls $r < 1$ stay inside (Lemma 16.2).

Entropy regularizer: KL Bregman balls



$h(x) = \sum_i x_i \log x_i$ (Lec 8). Bregman ball

$D_h(y, x_0) \le r$ around x_0.

Both geometries become anisotropic near the boundary — pinching toward the closer face.

Difference: the log barrier's local unit ball is *guaranteed* to stay inside (Dikin), while KL Bregman balls only do so for small enough radius. Self-concordance is what makes this guarantee uniform.

Global Barrier Inequality

Lemma 16.3 (Global barrier inequality). Let Φ be a ν-self-concordant barrier for K. For every $x \in \mathrm{int}(K)$ and $y \in K$:

$$\boxed{\langle \nabla \Phi(x), y - x \rangle \;\leq\; \nu.}$$

Interpretation. The barrier gradient "pushes away from the boundary", and how hard it pushes is controlled by ν — uniformly over feasible directions.

Two technical ingredients.

- Local gradient bound: $\|\nabla \Phi\|_{x,*} \leq \sqrt{\nu}$.
- Convexity of Φ along the segment $[x, y]$: ϕ' is nondecreasing.

Why this matters. The barrier-gap theorem (next slide) bounds suboptimality by ν/t — this lemma is the key step.

Proof of Lemma 16.3: Global Barrier Inequality

Setup. $\phi(t) := \Phi(x + t(y - x))$ on $[0, 1)$. If $\phi'(0) \le 0$, immediate. Else: convexity of $\Phi \Rightarrow \phi'(t) \ge \phi'(0) > 0$ for $t \in [0, 1)$.

Local-norm bound on ϕ'. Cauchy-Schwarz in local norm + barrier definition:

$$|\phi'(t)| = |\langle \nabla\Phi(x + t(y - x)), y - x\rangle| \le \sqrt{\nu}\,\sqrt{\phi''(t)} \;\Rightarrow\; \phi''(t) \ge \frac{\phi'(t)^2}{\nu}.$$

Riccati-style integration. Divide by $\phi'(t)^2$ + integrate:

$$\frac{d}{dt}\frac{1}{\phi'(t)} = -\frac{\phi''(t)}{\phi'(t)^2} \le -\frac{1}{\nu} \quad \Rightarrow \quad 0 \le \frac{1}{\phi'(t)} \le \frac{1}{\phi'(0)} - \frac{t}{\nu}\ \forall t \in [0, 1).$$

Let $t \uparrow 1$. $\phi'(0) \le \nu$, i.e., $\langle \nabla\Phi(x), y - x\rangle \le \nu$. $\square$

Barrier Subproblem and the Central Path

Barrier subproblem. For $t > 0$, on $\operatorname{int}(K)$:

$$F_t(x) := t\,\langle c, x\rangle + \Phi(x), \qquad x(t) := \operatorname*{argmin}_{x \in \operatorname{int}(K)} F_t(x).$$

Why $x(t)$ is well-defined. $\nabla^2\Phi \succ 0 \Rightarrow F_t$ strictly convex. (Assume each F_t attains its minimum.)

Newton decrement of F_t. Since $\nabla^2 F_t = \nabla^2\Phi$:

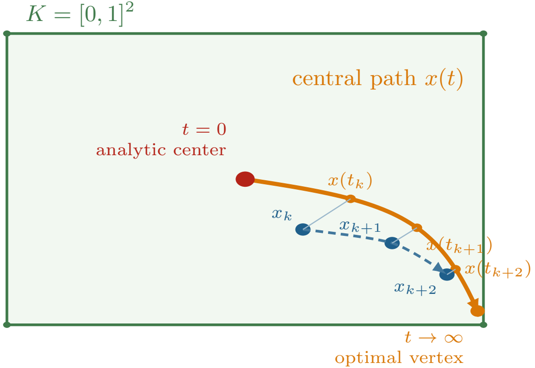
$$\lambda_{F_t}(x) = \|\nabla F_t(x)\|_{x,*} = \|tc + \nabla\Phi(x)\|_{x,*}.$$

Centrality condition. $x(t)$ on the central path $\Leftrightarrow$

$$\boxed{\lambda_{F_t}(x(t)) = 0 \;\Leftrightarrow\; tc + \nabla\Phi(x(t)) = 0.}$$

Geometric picture. As $t \to 0$: $x(t)$ approaches the *analytic center* (min of Φ). As $t \to \infty$: $x(t)$ approaches the *optimal face* of K.

Central Path



Central path $x(t)$ (orange) on $K = [0,1]^2$, moving from the analytic center toward the optimal vertex as t grows. Blue dots are the algorithmic iterates x_k (introduced later); each lies within a small δ-neighborhood of the corresponding central point.

Barrier Gap Theorem

Theorem 16.4 (Barrier gap). Let $x(t)$ be the central-path point at parameter $t > 0$. Then

$$\boxed{\langle c, x(t)\rangle - p^\star \;\le\; \frac{\nu}{t}.}$$

Proof. Fix $y \in K$. Centrality: $tc + \nabla\Phi(x(t)) = 0$, so

$$t\big(\langle c, x(t)\rangle - \langle c, y\rangle\big) = \langle \nabla\Phi(x(t)), y - x(t)\rangle.$$

Apply Lemma 16.3 (global barrier inequality):

$$\langle c, x(t)\rangle - \langle c, y\rangle \;\le\; \frac{\nu}{t}.$$

Take infimum over $y \in K$. $\square$

Algorithmic message. Reach ε-optimal by following $x(t)$ until $t \ge \nu/\varepsilon$. The question is *how cheaply* can we approximate $x(t)$ as t grows.

Note. Same argument extends to convex f_0: if $x(t)$ minimizes $tf_0 + \Phi$, then $f_0(x(t)) - p^\star \le \nu/t$.

Short-Step Primal Interior-Point Method

Algorithm (Short-Step IPM). Constants $0 < \delta \leq 1/10$. Strictly feasible $x_0 \in \mathrm{int}(K)$, $t_0 > 0$ with $\lambda_{F_{t_0}}(x_0) \leq \delta$.

For $k = 0, 1, 2, \ldots$:

1. **If** $t_k \geq 2\nu/\varepsilon$: **return** x_k.
2. **Increase** t**:** $t_{k+1} := t_k\left(1 + \dfrac{\delta}{\sqrt{\nu}}\right)$.
3. **One Newton step on** $F_{t_{k+1}}$**:** $d_k := -(\nabla^2\Phi(x_k))^{-1}(t_{k+1}c + \nabla\Phi(x_k))$.
4. $x_{k+1} := x_k + d_k$.

Two key mechanisms.

- **Increasing** t enlarges the decrement by at most $O(\sqrt{\nu})$ (Lemma 16.5).
- **One Newton step** contracts the decrement quadratically (Lemma 16.6 / Lec 15).

Newton neighborhood. The invariant $\lambda_{F_{t_k}}(x_k) \leq \delta$ defines a δ-*neighborhood* of the central path that the algorithm preserves.

Mechanism 1: Changing t Enlarges Decrement by $O(\sqrt{\nu})$

Lemma 16.5. Fix $x \in \mathrm{int}(K)$, $\eta > 0$, $t^+ := (1+\eta)t$. Then

$$\lambda_{F_{t^+}}(x) \;\le\; (1+\eta)\lambda_{F_t}(x) + \eta\sqrt{\nu}.$$

Proof. Hessian is independent of t: $\nabla^2 F_s(x) = \nabla^2 \Phi(x)$, so $\|\cdot\|_{x,*}$ doesn't depend on s.

Gradient change. $\nabla F_{t^+}(x) = \nabla F_t(x) + \eta tc$, and $tc = \nabla F_t(x) - \nabla\Phi(x)$. Triangle inequality + barrier definition:

$$\|tc\|_{x,*} \le \|\nabla F_t(x)\|_{x,*} + \|\nabla\Phi(x)\|_{x,*} \le \lambda_{F_t}(x) + \sqrt{\nu}.$$

Combine.

$$\lambda_{F_{t^+}}(x) \le \lambda_{F_t}(x) + \eta\|tc\|_{x,*} \le (1+\eta)\lambda_{F_t}(x) + \eta\sqrt{\nu}. \ \square$$

Specialize $\eta = \delta/\sqrt{\nu}$: $\lambda_{F_{t^+}}(x) \le (1+\delta/\sqrt{\nu})\delta + \delta = O(\delta)$.

Mechanism 2: One Newton Step Recenters

Lemma 16.6 (Newton recentering). Let $t > 0$ and $x \in \operatorname{int}(K)$ with $\lambda_{F_t}(x) < 1$. Let x^+ be the full Newton step for minimizing F_t. Then

$$\boxed{x^+ \in \operatorname{int}(K), \qquad \lambda_{F_t}(x^+) \le \left(\frac{\lambda_{F_t}(x)}{1 - \lambda_{F_t}(x)}\right)^2.}$$

Proof. Two ingredients.

- $F_t = t\langle c, x\rangle + \Phi$ is self-concordant (sum rule, Prop. 15.2, plus $\langle c, x\rangle$ is affine).
- $\|x^+ - x\|_x = \lambda_{F_t}(x) < 1$, so Dikin ellipsoid inclusion (Lemma 16.1) puts $x^+ \in \operatorname{int}(K)$.

The full-step decrement recursion (Theorem 15.9) then gives the displayed bound. □

Takeaway. Provided we stay in the Newton neighborhood $\lambda < 1$, one full Newton step *squares* the decrement (up to the $1 - \lambda$ correction).

Short-Step Theorem (Schedule + Invariant)

Theorem 16.7. $\nu \geq 1$, $0 < \delta \leq 1/10$, $t_0 > 0$ with $\lambda_{F_{t_0}}(x_0) \leq \delta$. With $t_{k+1} = t_k(1 + \delta/\sqrt{\nu})$ + one full Newton step on $F_{t_{k+1}}$:

$$\lambda_{F_{t_k}}(x_k) \leq \delta \; \forall k, \qquad \langle c, x_N \rangle - p^\star \;\leq\; \tfrac{2\nu}{t_N}.$$

Corollary 16.8 (Iteration complexity). Reach $t_N \geq 2\nu/\varepsilon$ in

$$N \;=\; O\Big(\sqrt{\nu}\, \log \tfrac{\nu}{\varepsilon t_0}\Big) \quad \text{Newton steps.}$$

Slogan. $O(\sqrt{\nu}\,\log(\nu/\varepsilon))$ Newton steps to ε-optimality. The $\sqrt{\nu}$ comes from the t-schedule; the log from t growing geometrically.

Proof of Theorem 16.7 (1/2): Decrement Invariant

Inductive hypothesis. $\lambda_{F_{t_k}}(x_k) \le \delta$.

Step 1 (apply Lemma 16.5). With $\eta = \delta/\sqrt{\nu}$:

$$\lambda_{F_{t_{k+1}}}(x_k) \le \left(1 + \tfrac{\delta}{\sqrt{\nu}}\right)\delta + \delta \le 2\delta + \tfrac{\delta^2}{\sqrt{\nu}} \le \delta_{\mathrm{mid}} := 2\delta + \delta^2.$$

Step 2 (key numerical bound). For $\delta \le 1/10$: $\delta_{\mathrm{mid}} < 1$, and

$$\frac{\delta_{\mathrm{mid}}}{1-\delta_{\mathrm{mid}}} = \frac{\delta(2+\delta)}{1-2\delta-\delta^2} \le \frac{210}{79}\delta \;\Rightarrow\; \left(\frac{\delta_{\mathrm{mid}}}{1-\delta_{\mathrm{mid}}}\right)^2 \le \left(\frac{210}{79}\right)^2 \delta^2 \le \delta.$$

(Last bound uses $\delta \le 1/10 < (79/210)^2$.)

Step 3 (apply Lemma 16.6). Full Newton step on $F_{t_{k+1}}$ gives

$$\lambda_{F_{t_{k+1}}}(x_{k+1}) \le \left(\frac{\delta_{\mathrm{mid}}}{1-\delta_{\mathrm{mid}}}\right)^2 \le \delta. \quad \text{Invariant preserved. } \checkmark$$

Proof of Theorem 16.7 (2/2): Objective Accuracy

Goal. Bound $\langle c, x_N\rangle - p^\star$ in terms of t_N.

Step 1 (distance to exact central point $x(t_N)$). Decrement cert (Thm 15.8) + lower SC at minimizer:

$$\omega(\|x_N - x(t_N)\|_{x(t_N)}) \le F_{t_N}(x_N) - F_{t_N}(x(t_N)) \le \omega^*(\delta).$$

Monotonicity of ω + elementary $\omega^{-1}(\omega^*(\delta)) \le \delta/(1-\delta)$:

$$\|x_N - x(t_N)\|_{x(t_N)} \ \le \ \tfrac{\delta}{1-\delta}.$$

Step 2 (cross-link). Centrality: $t_N c = -\nabla\Phi(x(t_N))$. Cauchy-Schwarz in local norm:

$$t_N(\langle c, x_N\rangle - \langle c, x(t_N)\rangle) \le \|\nabla\Phi(x(t_N))\|_{x(t_N),*}\|x_N - x(t_N)\|_{x(t_N)} \le \sqrt{\nu}\cdot\tfrac{\delta}{1-\delta}.$$

Step 3 (combine with barrier gap). Thm 16.4 + Step 2:

$$\langle c, x_N\rangle - p^\star \le \tfrac{\nu}{t_N} + \tfrac{\sqrt{\nu}}{t_N}\cdot\tfrac{\delta}{1-\delta} \le \tfrac{2\nu}{t_N},$$

using $\nu \ge 1$, $\delta \le 1/10 \Rightarrow \delta/(1-\delta) \le 1$. □

Iteration Complexity: Counting Newton Steps

Corollary 16.8 proof. The schedule $t_{k+1} = t_k(1 + \delta/\sqrt{\nu})$ is geometric:

$$t_N = t_0\left(1 + \frac{\delta}{\sqrt{\nu}}\right)^N.$$

Lower bound on $\log(1 + \delta/\sqrt{\nu})$**.** Since $0 < \delta/\sqrt{\nu} \le 1$:

$$\log\left(1 + \frac{\delta}{\sqrt{\nu}}\right) \ge \frac{\delta}{2\sqrt{\nu}}.$$

Solve for N. Want $t_N \ge 2\nu/\varepsilon$, i.e., $N \log(1 + \delta/\sqrt{\nu}) \ge \log(2\nu/(\varepsilon t_0))$. So

$$\boxed{N \le \frac{2}{\delta}\sqrt{\nu}\,\log\left(\frac{2\nu}{\varepsilon t_0}\right) + 1 = O\left(\sqrt{\nu}\,\log\frac{\nu}{\varepsilon t_0}\right).}$$

Specializing.

- LP: $\nu = m \Rightarrow O(\sqrt{m}\log(m/(\varepsilon t_0)))$.
- SDP: $\nu = n \Rightarrow O(\sqrt{n}\log(n/(\varepsilon t_0)))$.

Per-step cost. One linear solve with Hessian $\nabla^2\Phi(x_k)$. For dense LP: $O(mn^2 + n^3)$.

Feasible-Start: Two-Phase Method

Issue. Thm 16.7 needs $\lambda_{F_{t_0}}(x_0) \leq \delta$. A given $x_{\mathrm{feas}} \in \mathrm{int}(K)$ may be far from the central path.

Two-phase fix.

1. **Centering:** Newton on Φ alone until $\lambda_\Phi(x_{\mathrm{cen}}) \leq 1/20$.
2. **Path-following:** set $t_0 := 1/(20R_c)$, run short-step IPM with $\delta = 1/10$ until $t_N \geq 2\nu/\varepsilon$.

$R_c := \sup_K \langle c, x\rangle - \inf_K \langle c, x\rangle$ (objective range).

Theorem 16.9. The output is strictly feasible with $\langle c, x_N\rangle - p^\star \leq \varepsilon$, and total Newton-step count

$$O\Big(\underbrace{\Phi(x_{\mathrm{feas}}) - \Phi^\star + 1}_{\text{centering}} + \underbrace{\sqrt{\nu}\,\log(1 + \nu R_c/\varepsilon)}_{\text{path-following}}\Big).$$

Proof of Theorem 16.9

Step 1 (centering cost). Dikin inclusion supplies the (DS) assumption Lec. 15 needs. Newton synthesis (Thm 15.10) on Φ alone reaches

$$\lambda_\Phi(x_{\mathrm{cen}}) \le \tfrac{1}{20} \quad \text{in} \quad O\big(\Phi(x_{\mathrm{feas}}) - \Phi^\star + 1\big) \text{ steps.}$$

Step 2 ($\|c\|_{x_{\mathrm{cen}},*} \le R_c$ via Dikin). If $\|h\|_{x_{\mathrm{cen}}} < 1$, then $x_{\mathrm{cen}} \pm h \in K$ (Dikin), so $2|\langle c, h\rangle| \le R_c$. Taking $\sup_{\|h\|_{x_{\mathrm{cen}}} \le 1}$: $\|c\|_{x_{\mathrm{cen}},*} \le R_c$.

Step 3 ($t_0 = 1/(20R_c)$ enters the $\delta = 1/10$ neighborhood).

$$\lambda_{F_{t_0}}(x_{\mathrm{cen}}) \le \underbrace{\lambda_\Phi(x_{\mathrm{cen}})}_{\le 1/20} + \underbrace{t_0\|c\|_{x_{\mathrm{cen}},*}}_{\le 1/20} \le \frac{1}{10} = \delta. \checkmark$$

Step 4 (path-following + feasibility). Cor. 16.8 from (x_{cen}, t_0) with $\delta = 1/10$:

$$N_{\mathrm{pf}} = O\Big(\sqrt{\nu}\log(1 + \nu/(\varepsilon t_0))\Big) = O\Big(\sqrt{\nu}\log(1 + \nu R_c/\varepsilon)\Big).$$

Strict feasibility: each Newton step has local length < 1 (Dikin). Thm 16.7 $\Rightarrow$ $\langle c, x_N\rangle - p^\star \le 2\nu/t_N \le \varepsilon$. $\square$

Specialization: Dense LP Arithmetic Cost

Corollary 16.10 (Dense LP arithmetic cost). LP $\min\{c^\top x : Ax \le b\}$, $K = \{x : Ax \le b\}$ bounded, full-dim, strictly feasible x_{feas}. Log barrier $\Phi(x) = -\sum_i \log(b_i - a_i^\top x)$, $\nu = m$. After

$$O\Big(\Phi(x_{\text{feas}}) - \Phi^\star + \sqrt{m}\log(1 + mR_c/\varepsilon)\Big)$$

Newton steps, $c^\top x_N - p^\star \le \varepsilon$.

Per-step cost. $\nabla^2\Phi(x) = A^\top \operatorname{diag}((b - Ax)^{-2})A$. Form Hessian $O(mn^2)$ + solve $n \times n$ system $O(n^3) = O(mn^2 + n^3)$.

Total arithmetic. $O\big((\Phi(x_{\text{feas}}) - \Phi^\star + \sqrt{m}\log(1 + mR_c/\varepsilon))(mn^2 + n^3)\big)$.

What this is and isn't. *Feasible-start arithmetic* bound, not a full polynomial-time LP theorem. Two quantities are not controlled by input bit length: the centering cost $\Phi(x_{\text{feas}}) - \Phi^\star$ and the objective scale R_c/ε.

Comparison with the Ellipsoid Method

What Thm 16.9 / Cor 16.10 are. *Feasible-start arithmetic* bounds, not full polynomial-time LP theorems. Two quantities are not controlled by input bit length: the centering cost $\Phi(x_{\mathrm{feas}}) - \Phi^\star$ and the objective scale R_c/ε.

Total operations for LP (dense data, m constraints in $\mathbb{R}^n$).

- **Ellipsoid:** $O(n^2 \log(R/r))$ separation-oracle calls. Each oracle call $O(mn)$ to find a violated row + $O(n^2)$ rank-1 update. Total: $O\big((mn + n^2)\, n^2 \log(R/r)\big)$.
- **Short-step barrier:** $O(\sqrt{m} \log(mR_c/\varepsilon))$ Newton steps, each forming and solving $\nabla^2\Phi$ in $O(mn^2 + n^3)$. Total: $O\big((mn^2 + n^3)\sqrt{m} \log(mR_c/\varepsilon)\big)$.

Summary

Self-concordant barrier. Function on $\operatorname{int}(K)$ that:

- is self-concordant + positive-definite Hessian + boundary blow-up;
- satisfies the local gradient bound $\|\nabla\Phi(x)\|_{x,*} \le \sqrt{\nu}$.

ν is the *barrier parameter* — the complexity input for path-following.

Two structural facts.

- **Dikin ellipsoid inclusion:** $\{y : \|y - x\|_x < 1\} \subseteq \operatorname{int}(K)$ (automatic domain safety).
- **Global barrier inequality:** $\langle \nabla\Phi(x), y - x\rangle \le \nu$.

Central path. $x(t) = \operatorname{argmin}\, t\langle c, x\rangle + \Phi(x)$. Barrier gap: $\langle c, x(t)\rangle - p^\star \le \nu/t$.

Short-step IPM. $t_{k+1} = t_k(1 + \delta/\sqrt{\nu})$ + one Newton step per outer iteration. Total complexity: $O(\sqrt{\nu}\,\log(\nu/\varepsilon))$ **Newton steps.**

LP specialization. $\sqrt{m}$ outer iterations, $O(mn^2 + n^3)$ per-step linear algebra.

Next. Lecture 17: **primal-dual** IPM via perturbed KKT equations; handles initialization cleanly and is the implementation backbone of conic solvers.

Bibliographic Notes

Yurii Nesterov

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